Worcester State University Placement Test
Precalculus Study Guide

Disclaimer: *This study guide is intended to assist in preparation for the Precalculus Placement Test at Worcester State University by providing practice problems on the various topics covered on the test, as well as skills needed in all courses beyond MA 190: Precalculus. Please note that problems on the test will be different than those presented here. Answers to all problems appear at the end of this packet. If you require any assistance, free drop-in tutoring is available at the WSU Math Center (Sullivan 140) during the fall and spring semesters – stop by S-140 to see the current schedule. NOTE: Students may NOT use calculators during the test.*

Exponential Functions and Logarithms

Understanding the behavior and properties of these functions is essential in calculus as well as in many areas of science.

1. Sketch the graphs of the following functions. State the domain and range of each in interval notation.

   a) $y = 2^x$

   b) $y = \left(\frac{1}{3}\right)^x$

   c) $y = 2^{x-1} + 4$

   d) $y = -\left(\frac{1}{3}\right)^{x+3}$

2. Simplify the expression $\left(\frac{x^2 y^{-4}}{x^{-5}}\right)^3$. Leave no negative exponents in your answer.

3. Evaluate the following values.

   a) $\log_3(81)$

   b) $\log_7(\frac{1}{49})$

   c) $\ln\left(\frac{1}{e^2}\right)$

   d) $\log_6 1$

   e) $\log_{\frac{1}{2}}(8)$

   f) $\log_{27}(3)$

4. Find the domain of $f(x) = 5\log_4(3x + 16) - 1$.

5. Rewrite $6\log(y) - 3\log(x^2)$ as a single logarithm.

6. Use properties of logarithms to expand $\log_5\left(\frac{x+3}{x^4}\right)$ as much as possible.

7. Use properties of logarithms to expand $\log_2\left(\frac{\sqrt{2}}{4}\right)$ as much as possible.

8. Solve the following equations.

   a) $5 \cdot 3^x = 135$

   b) $5e^{4-3x} - 4 = 36$

   c) $x^2e^x + xe^x - 2e^x = 0$

   d) $4 + 3\log(2x) = 16$

   e) $\log_5(x-2) + \log_5(x+1) = \log_5(4)$

   f) $\log_3(x+15) - \log_3(x-1) = 2$

Trigonometric Functions

The six trigonometric functions (sine, cosine, tangent, secant, cosecant, cotangent) can be defined using ratios of side lengths of right triangles or using points on the unit circle. Both approaches are important to understand in order to apply trigonometry in all situations in which it is required in calculus.

9. Find the exact value for each of the following.

   a) $\sin\left(\frac{\pi}{3}\right)$

   b) $\cos\left(-\frac{\pi}{6}\right)$

   c) $\tan\left(\frac{5\pi}{6}\right)$

   d) $\sec\left(\frac{5\pi}{3}\right)$

   e) $\csc\left(\frac{3\pi}{2}\right)$

   f) $\cot(-3\pi)$

10. Simplify the following trigonometric expressions.

    a) $\cos^{-1}\left(\cos\left(-\frac{\pi}{4}\right)\right)$

    b) $\sin^2 x - (1-\cos x)(1+\cos x)$

    c) $\frac{\sin t}{1-\sin t} \cdot \frac{\sin t}{1+\sin t}$

11. Given that $\sin(t) = \frac{2}{5}$ and that $\frac{\pi}{2} < t < \pi$, find $\cos(t)$.

12. Given $\cos\theta = \frac{5}{9}$, sketch a right triangle that has acute angle θ and use it to find the other five trigonometric functions of θ.

13. Sketch one period of the function $f(x) = 3\sin\left(x + \frac{\pi}{4}\right)$.

14. Find the exact value of the following.

a) $\sin^{-1}\left(\sin\left(\dfrac{-3\pi}{7}\right)\right)$

b) $\sin^{-1}\left(\sin\left(\dfrac{9\pi}{11}\right)\right)$

15. Solve the following trigonometric equations. Find all solutions in the interval $[0, 2\pi)$.

a) $2\cos\theta + 3 = 2$

b) $\tan^2\theta = 3$

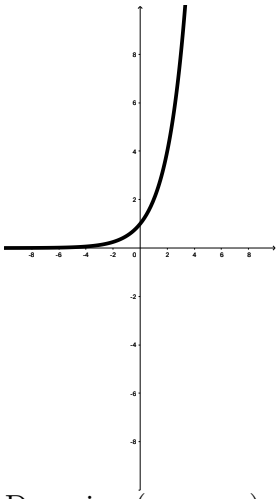
c) $2\sin^2\theta - \sin\theta = 0$

d) $2\sin^2\theta = 3(\cos\theta + 1)$

e) $\cos\theta - 2\cos\theta\sin\theta = 0$

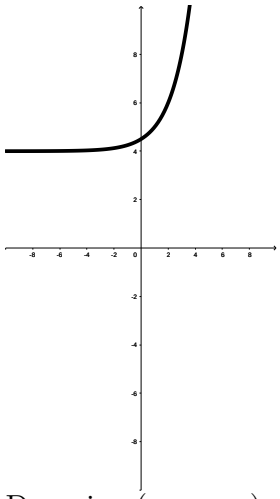
Answers

1. a)



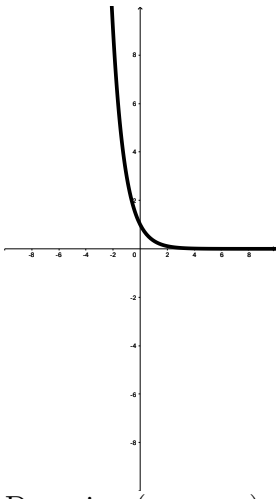
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

c)



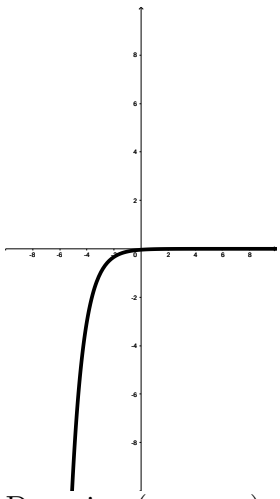
Domain: $(-\infty, \infty)$
Range: $(4, \infty)$

b)



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

d)



Domain: $(-\infty, \infty)$
Range: $(-\infty, 0)$

2. $\dfrac{x^{21}}{y^{12}}$

3. a) 4 b) -2 c) -2 d) 0 e) -3 f) $\dfrac{1}{3}$

4. $\left(-\dfrac{16}{3}, \infty\right)$

5. $\log\left(\dfrac{y^6}{x^6}\right)$

6. $\log_5(x+3) - 4\log_5(x)$

7. $-\dfrac{3}{2}$

8. a) 3 b) $\dfrac{\ln(8)-4}{-3}$ c) $-2, 1$ d) 5000 e) 3 f) 3

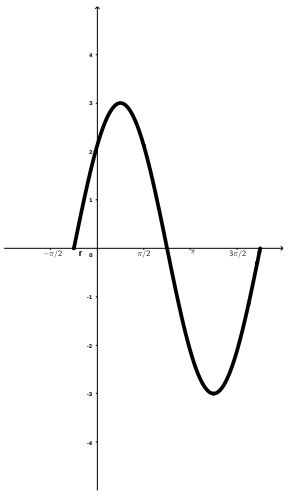
9. a) $\dfrac{\sqrt{3}}{2}$ c) $-\dfrac{1}{\sqrt{3}}$ e) -1

   b) $\dfrac{\sqrt{3}}{2}$ d) 2 f) Undefined

10. a) $\dfrac{3\pi}{4}$ b) 0 c) $\tan^2(t)$

11. $-\dfrac{\sqrt{21}}{5}$

12. $\sin\theta = \dfrac{2\sqrt{14}}{9}$, $\tan\theta = \dfrac{2\sqrt{14}}{5}$, $\sec\theta = \dfrac{9}{5}$, $\csc\theta = \dfrac{9}{2\sqrt{14}}$, $\cot\theta = \dfrac{5}{2\sqrt{14}}$

13. 

14. a) $\dfrac{-3\pi}{7}$ b) $\dfrac{2\pi}{11}$

15. a) $\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

    b) $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

    c) $\theta = 0, \pi, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

    d) $\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$

    e) $\theta = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$