

Worcester State University Placement Test Functions Study Guide

Disclaimer: *This study guide is intended to assist in preparation for the Functions Placement Test at Worcester State University by providing practice problems on the various topics covered on the test, as well as skills needed in all courses beyond MA 180: Introduction to Functions. Please note that problems on the test will be different than those presented here. Answers to all problems appear at the end of this packet. If you require any assistance, free drop-in tutoring is available at the WSU Math Center (Sullivan 140) during the fall and spring semesters – stop by S-140 to see the current schedule. NOTE: Students may NOT use calculators during the test.*

Function Basics

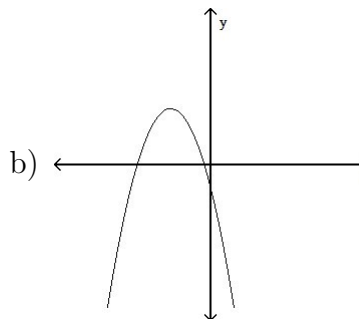
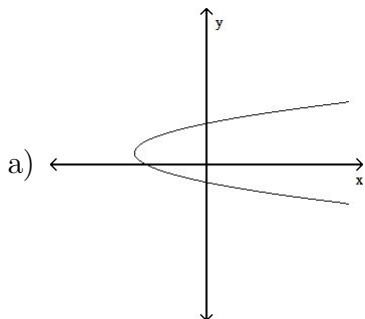
The concept of a function is used throughout college-level mathematics courses. Some important concepts relating to functions include determining whether a relation is a function, identifying a function's domain and range, computing average rates of change and difference quotients, transforming graphs of functions, and finding inverse functions. Problems in this section review these fundamental topics.

1. Find the domain and range of each relation below. Then determine whether each represents a function.

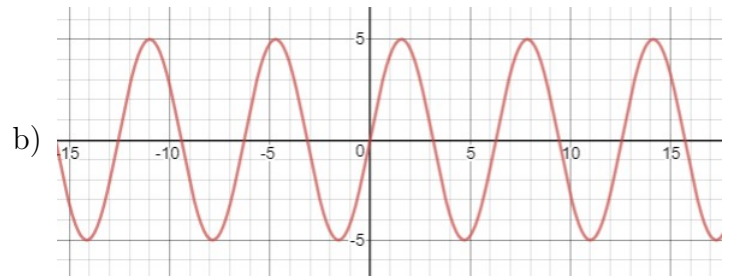
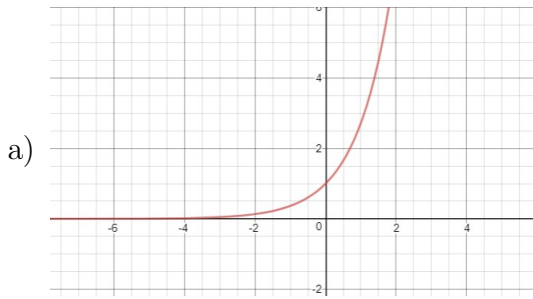
a) $\{(1, 3), (1, 5), (1, 4), (5, 7), (7, 6)\}$

b) $\{(1, 3), (2, 3), (3, 3), (4, 3)\}$

2. Determine if each of the graphs below represents y as a function of x .



3. Find the domain and range of the following functions.



4. For each of the functions below, (i) find the average rate of change of $f(x)$ on the interval $[1, 4]$, and (ii) find and simplify the difference quotient: $\frac{f(x+h) - f(x)}{h}$, where $h \neq 0$.

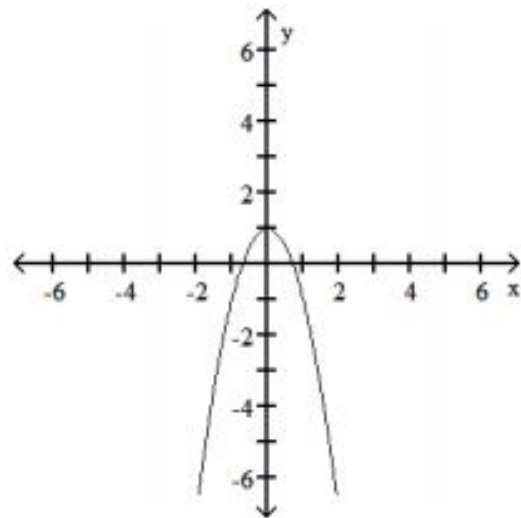
a) $f(x) = 8x - 4$

d) $f(x) = \sqrt{x}$

b) $f(x) = x^2 - 2x$

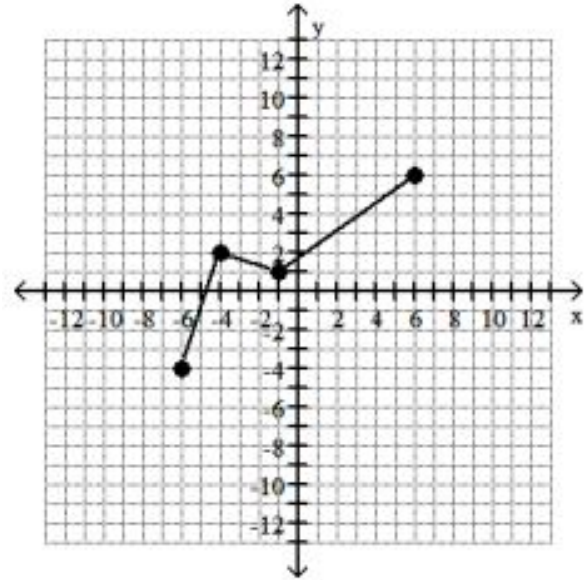
c) $f(x) = \frac{4}{x}$

5. The graph to the right shows a function $f(x)$ whose graph was formed by applying transformations to the graph of $y = x^2$. Find a formula for the function $f(x)$ shown in the graph.

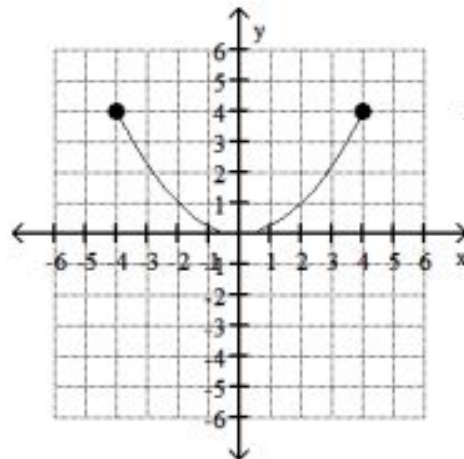


6. Find the function $f(x)$ that is graphed after the following transformations are applied to the graph of $y = |x|$. The graph is shifted right 3 units, stretched by a factor of 3, shifted vertically down 2 units, and finally reflected across the x -axis.

7. The graph of a function $y = f(x)$ is shown on the right. Use this to graph the function $g(x) = -2f(x)$.



8. The graph of a function $y = f(x)$ is shown on the right. Use this to graph the function $g(x) = -f(x + 2) - 1$.



9. Determine whether or not $f(x)$ and $g(x)$ are inverses of each other.

a) $f(x) = 9x - 7$, $g(x) = \frac{x + 9}{7}$

b) $f(x) = 3x + 9$, $g(x) = \frac{1}{3}x - 3$

10. Find the inverse function, $f^{-1}(x)$, for each given function $f(x)$.

a) $f(x) = \frac{x + 1}{x}$

b) $f(x) = 6 - 0.5x$

c) $f(x) = \frac{3x}{x - 1}$

Solving Equations

There are many different types of equations (e.g. linear, quadratic, rational, radical, etc.); each type requires algebraic techniques to solve. This section provides a collection of equations to be solved. Be sure you know how to find all solutions and when to check for extraneous ones!

11. $3 - 2(x - 1) = x - 10$

12. $\frac{t}{2} - \frac{t}{7} = 1$

13. $(u - 4)^2 - 10 = 15$

14. $2z^2 + 7z - 4 = 0$

15. $y^2 + 2y = 7$

16. $3w^2 - 7w + 1 = 0$

17. $x^2 + \frac{2}{3}x + 1 = \frac{4}{3}$

18. $\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$

19. $3x^4 = 27x^2$

20. $\sqrt{x} - 3 = x - 9$

21. $4|1 - \frac{3}{4}x| + 7 = 10$

22. $3x^{\frac{3}{4}} - 6 = 0$

23. Solve $xy^2 - xz = z + 4xy - 6$ for x

Word Problems

Word problems are solved by translating a written description to an equation, solving the equation, and checking that the solution(s) are appropriate given the context of the problem. Find and solve equations which solve the following word problems.

24. You have \$60 saved and your sister has \$120 saved. You are saving \$7 per week and your sister is saving \$5 per week. How long will it be until you have the same amount of money?
25. A rectangular swimming pool measures 40 feet by 60 feet. It is surrounded by a path of uniform width around the four edges. The perimeter of the rectangle formed by the pool and the surrounding path is 280 feet. Determine the width of the path.
26. The selling price of a refrigerator is \$584. If the markup is 25% of the dealer's cost, what is the dealer's cost of the refrigerator?
27. A rocket carrying fireworks is launched from a hill 80 feet above a lake. The rocket will fall into the lake after exploding at its maximum height. The rocket's height above the surface of the lake is given by $h(t) = -16t^2 + 64t + 80$, where t is time in seconds after the rocket is launched.
- When will the rocket explode?
 - When will the rocket land in the lake?
28. You have 300 yards of fence and you need to fence a rectangular region that borders a river. The river provides a natural border, so you do not need to fence the side along the river. What is the maximum area that you can fence?

Answers

1. a) Domain: $\{1, 5, 7\}$, range $\{3, 4, 5, 6, 7\}$, not a function

b) Domain $\{1, 2, 3, 4\}$, range $\{3\}$, function

2. a) No

b) Yes

3. a) Domain $(-\infty, \infty)$, range $(0, \infty)$

b) Domain $(-\infty, \infty)$, range $[-5, 5]$

4. a) (i) 8, (ii) 8

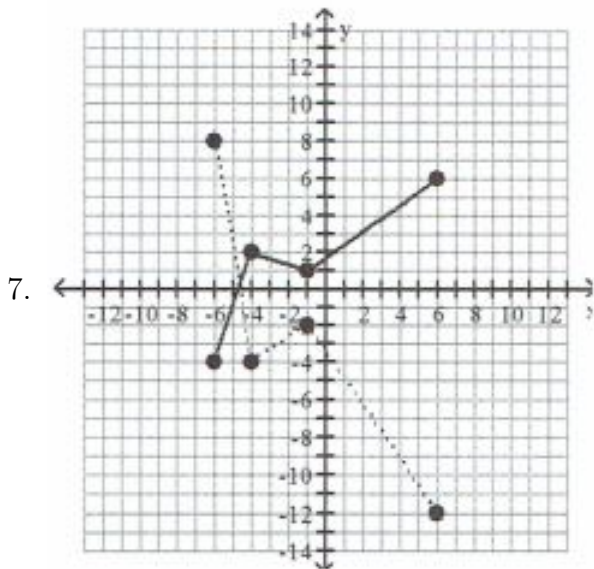
c) (i) -1 , (ii) $\frac{-4}{x(x+h)}$

b) (i) 3, (ii) $2x - 2 + h$

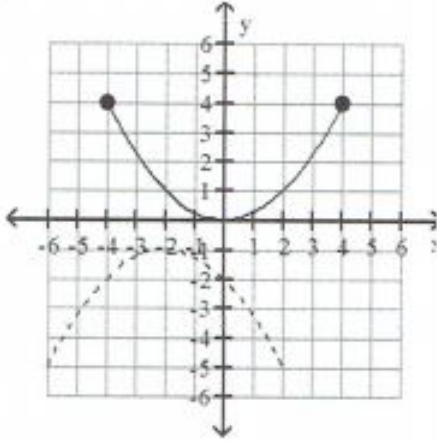
d) (i) $\frac{1}{3}$, (ii) $\frac{1}{\sqrt{x+h} + \sqrt{x}}$

5. $y = -2x^2 + 1$

6. $y = -3|x - 3| + 2$



8.



9. a) No

b) Yes

10. a) $f^{-1}(x) = \frac{1}{x-1}$

b) $f^{-1}(x) = 12 - 2x$

c) $f^{-1}(x) = \frac{x}{x-3}$

11. $x = 5$

16. $w = \frac{7 \pm \sqrt{37}}{6}$

20. $x = 9$

12. $t = \frac{14}{5}$

17. $x = \frac{1}{3}, -1$

21. $x = \frac{1}{3}, \frac{7}{3}$

13. $u = -1, 9$

18. $x = \frac{4}{3}$

22. $x = 2\sqrt[3]{2}$

14. $x = \frac{1}{2}, -4$

15. $y = -1 \pm 2\sqrt{2}$

19. $x = 0, 3, -3$

23. $x = \frac{z-6}{y^2-z-4y}$

24. Let x be the number of weeks that have passed

Equation: $60 + 7x = 120 + 5x$

Solution: 30 weeks

25. Let x be the width of the path

Equation: $2(60 + 2x) + 2(40 + 2x) = 280$

Solution: The path is 10 feet wide.

26. Let c be the dealer's cost

Equation: $c + 0.25c = 584$

Solution: The dealer's cost is \$467.20.

27. a) 2 minutes after it is launched

b) $0 = -16t^2 + 64t + 80$

The rocket lands in the lake 5 seconds after it is launched.

28. Let w represent the width of the river.

$A = -2w^2 + 300w$

Solution: 11,250 square feet.